Supplementary of SVD-free Convex-Concave Approaches for Nuclear Norm Regularization

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1 Proof of Theorem 2

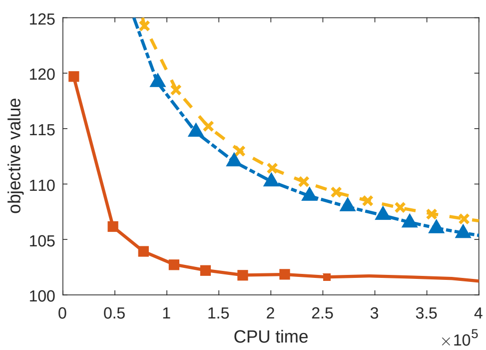

Figure 1: Results of robust low-rank matrix approximation

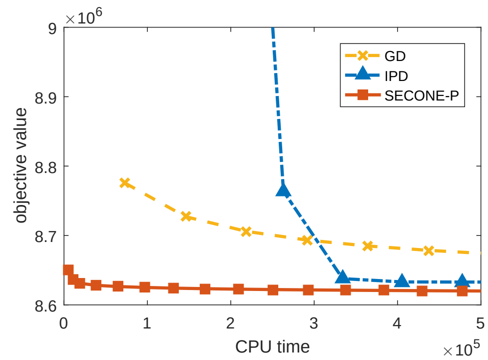

Figure 2: Results of sparse and low-rank link prediction

Table 1: Statistics for matrix approximation

Method	c_1	c_2	T	Total CPU time
SECONE	$1e9$	10	36000	$4.05e5$
PGD	$1e9$		500	$4.10e5$
GD	$1e9$		500	$3.93e5$

Table 2: Statistics for link prediction

Method	c_1 or θ	c_2	T	Total CPU time
SECONE	1	$1e\ \ 5$	15500	$5.02e5$
IPD	0.01		450	$5.13e5$
GD	1		420	$5.10e5$

is insensitive to λ. As can be seen, SECONE decreases much faster than GD and PGD. This is as expected as SECONE is SVD-free and time-efficient, which is also convinced by the statistics shown in Table 1. As can be seen, each iteration of SECONE takes much less time than other two methods.

2.2 Sparse and Low-rank Link Prediction

Following the setting in [Richard *et al.*, 2012], we perform experiments on the Facebook100 dataset which contains the friendship relations between students. We select a single university with $41,554$ students and keep only the 10% users with the highest degree (e.g. $m = n = 4155$). We flip 15% of randomly chosen entries and the goal is to learn a sparse and low-rank matrix from the noisy adjacency matrix Y.

We compare Algorithm 3 (SECONE-P) with subgradient descent (GD) and Incremental Proximal Decent (IPD), which is an iterative algorithm designed for the above problem but with no theoretical guarantees [Richard *et al.*, 2012]. The step sizes in SECONE-P and GD are set in the same way as in Section 2.1. The parameter θ of IPD is searched in the range of $f10\ ^3, 10\ ^2, \ldots, 10g$.

In Fig. 2, we plot objective value versus the running time when $\lambda = 8$ and $\gamma = 0.4$. As can be seen, SECONE-P converges much faster than other methods, and GD performs the worst. The statistics of different methods are shown in Table 2. Again, the running time per iteration of SECONE-P is much smaller than other methods.

References

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